Signal To Thermal Noise Ratios

Dallas Lankford, 1/31/09, rev. 7/23/2011

I have been studying thermal noise of small loop antennas for over 20 years, and I have still not understood it as well as I would like. So here we go again.

The derivations which follow are mostly variations of Belrose's classical derivation for ferrite rod loop antennas, "Ferromagnetic Loop Aerials," **Wireless Engineer**, February 1955, 41– 46.

The signal voltage e_s in volts for a one turn loop of area A in meters and a signal of wavelength λ for a given radio wave is

$$e_s = [2\pi A\, E_s / \lambda]\, COS(\theta)$$

where E_s is the signal strength in volts per meter and θ is the angle between the plane of the loop and the radio wave.

It is well known that if an omnidirectional antenna, say a short whip, is attached to one of the output terminals of the loop and the phase difference between the loop and vertical and the amplitude of the whip are adjusted to produce a cardioid patten, then this occurs for a phase difference of about 90 degrees and a whip amplitude equal to the amplitude of the loop, and the signal voltage in this case is

$$e_s = [2\pi A\, E_s / \lambda]\, [1 + COS(\theta)] .$$

Notice that the maximum signal voltage of the cardioid antenna is twice the maximum signal voltage of the loop (or vertical) alone. A flag antenna is a one turn loop antenna with a resistance of several hundred ohms inserted at some point into the one turn. With a rectangular turn, with the resistor appropriately placed and adjusted for the appropriate value, the flag antenna will often generate a cardioid pattern. The exact mechanism by which this occurs is not given here. Nevertheless, based on measurements, the flag antenna signal voltage is approximately the same as the cardioid pattern given above. The difference between an actual flag and the cardioid pattern above is that an actual flag pattern is not a perfect cardioid for some cardioid geometries and resistors. In general a flag antenna pattern is a limaçon with signal voltage given by

$$e_s = [2\pi A\, E_s / \lambda]\, [1 + k\, COS(\theta)]$$

where k is a constant, say 0.90 or 1.1 for a "poor" flag, or between 0.99 and 1.01 for a "good" flag. This has virtually no effect of the maximum signal pickup, but can have a significant effect on the null depth.

Johnson – Nyquist noise, the thermal output noise voltage e_n for a resistor of resistance r, is

$$e_n = \sqrt{\{4kTrB\}}$$

where $k = 1.37 \times 10^{-23}$ is Boltzmann's constant, T is the absolute temperature (usually taken as 290), and B is the bandwidth in Hertz. The theory and experimental verification of this formula is found in J. Johnson, "Thermal Agitation of Electricity in Conductors", Phys. Rev. 32, 97 (1928) – the experiment, and H. Nyquist, "Thermal Agitation of Electric Charge in Conductors", Phys. Rev. 32, 110 (1928) – the theory.

For a loop antenna with inductor resistance r in series with a resistor R (which is a flag antenna when $R \neq 0$)

$$e_n = \sqrt{\{4kT\sqrt{(r + R)}B\}}$$

When R = 0, and the loop is rotated so that the signal is maximum, the open circuit signal to noise ratio is approximately

$S/tN = e_s / e_n = [2\pi A\, E_s / \lambda] / \sqrt{\{4kTrB\}} = [166Af / \sqrt{\{rB\}}]E_s$,

where L is the loop coil inductance.

For a flag antenna rotated so the the signal is maximum, the signal to noise ratio is approximately

$S/tN = e_s / e_n = 2[2\pi A\, E_s / \lambda] / \sqrt{\{4kT(r + R_{flag})B\}} = [166Af / \sqrt{\{(r + R_{flag})B\}}]E_s$,

where R_{flag} ($\neq 0$) is the terminating resistance of the flag antenna.

As can be seen, a broadband (non-resonant) loop antenna generally has a greater signal to thermal noise ratio than a flag antenna when the two have the same loop area. As a matter of fact, for a broadband loop antenna, r is usually so small that the thermal noise of a broadband loop antenna cannot be measured.

7/22/2011: My understanding of signal to thermal noise ratios was greatly improved during the past few as described in my new article in The Dallas Files, "Receiving Antenna Thermal Noise And Preamplifier Noise." Elsewhere the article title has been "Receiving Antenna Thermal Noise And Small Signal Amplifier Noise."